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TECHNICAL NOTE

An improved method for smoothing approximate exchange areas

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INTRODUCTION

Radiative heat transfer analyses in enclosures are often carried out using the zone method [1]. Fundamental to this method are the closely related concepts of view factors and direct exchange areas (DEAs). The view factor F_{ij} is the fraction of radiation emitted by surface i that is directly incident upon surface j. The DEA s_is_j is the view factor F_{ij} multiplied by the area, A_i, of surface i. The DEAs can be calculated from the formula

$$s_is_j = \int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} \mathrm{d}A_j\,\mathrm{d}A_i \qquad (1)$$

where r is the distance between general points on the two surfaces and θ_i and θ_j are the angles between the line joining the two general points and the normals to the surfaces.

Since all the radiation emitted by a surface must go somewhere the sum of all the view factors for a particular surface is unity. It follows that the sum of all the DEAs for a surface is its area. This is called the conservation condition and may be written as

$$\sum_j s_is_j = A_i. \qquad (2)$$

Furthermore, interchanging i and j does not affect the value of the integral in equation (1) and so the DEAs are symmetric, that is

$$s_is_j = s_js_i. \qquad (3)$$

The complex nature of the integral formula in equation (1) means that numerical evaluation is essential. The calculated DEAs are therefore only approximations to the correct values and so may not satisfy both the conservation and symmetry conditions. If the DEAs are not conservative then the sum of all the calculated surface fluxes will be non-zero. If the DEAs are not symmetric the calculated radiant heat flow from surface i to surface j will be different from the flow from surface j to surface i.

SMOOTHING THE APPROXIMATE DEAs

Smoothing is the process of taking a set of approximate DEAs which do not satisfy equations (2) and (3) and making them symmetric and conservative. A set which is not symmetric is easily made symmetric by setting s_is_j', the modified value of s_is_j, equal to the average of the original values of s_is_j and s_js_i. Achieving conservation is much harder.

An iterative smoothing method is described in [2]. The essence of this method is to take each surface in turn and determine the average error in the conservation equation (2), namely

$$D_i = \frac{A_i - \sum s_is_j}{m_i} \qquad (4)$$

where m_i is the number of non-zero DEAs for surface i. The DEAs for surface i are then modified by adding D_i to all the non-zero ones making them conservative. If $D_i < 0$ some of these modified DEAs may be negative. If this happens m_i is decreased, D_i is recalculated and the original DEAs (except for those which became negative) are modified using the new value of D_i. The modification of the DEAs for surface i destroys the symmetry. This is restored by

$$s_js_i' = s_is_j'. \qquad (5)$$

This, however, will mean that the conservation property of the DEAs for other surfaces no longer holds. The process is iterated, working through the rows in the same order and always using the most recently calculated values. It is found that the method converges rapidly.

A TEST PROBLEM

The method from [2], outlined above, was used to smooth a set of approximate DEAs for a unit cube containing a centrally placed cylindrical pipe of radius 0.0318. The geometrical symmetry of this enclosure means that the values of the DEAs from each of the two end walls to the pipe should be the same. Similarly, the DEAs from the other four walls to the pipe should be the same. The approximate DEAs were calculated using a Monte Carlo algorithm [3] which, because of its stochastic nature, gives different values for these DEAs. The values of the original and smoothed DEAs are given in Table 1.

Smoothing should improve the accuracy of the approximate DEAs. However, it can be seen from Table 1 that the original values of the DEAs from the end walls to the pipe are close together, but smoothing pushes these values much further apart. Smoothing also leaves the DEA from wall 2 considerably different from the DEAs for the other three walls.

AN IMPROVED SMOOTHING ALGORITHM

The test problem illustrates the main shortcoming of this smoothing method. The modification of the original DEAs does not take into account their size. Each non-zero DEA for surface i is modified by the same amount. The DEAs

NOMENCLATURE

A area of a surface
D average error in direct exchange areas for a surface
F view factor from one surface to another
m number of non-zero direct exchange areas for a surface
r distance between general points in two surfaces
s_is_j direct exchange area from surface i to surface j.

Greek symbols
δ error in direct exchange area sum for a surface
θ angle between the normal to a surface and the path between general points in two surfaces.

Subscripts
i, j surface numbers.

Superscripts
$'$ Modified value.

Table 1. DEAs from the ends and the other walls to the pipe

	End 1	End 2	Wall 1	Wall 2	Wall 3	Wall 4
Original	0.0300	0.0306	0.0337	0.0355	0.0345	0.0360
Smoothed	0.0309	0.0288	0.0343	0.0361	0.0344	0.0353

from the walls to each other are all around 0.2, whereas from a wall to the pipe the DEA is approximately 0.03. The modification to these values should take into account their relative sizes. This can be achieved by altering the algorithm in the following way.

Determine the error in the conservation condition for surface i, namely

$$\delta_i = A_i - \sum s_is_j. \tag{6}$$

Then modify each DEA for surface i in a manner proportional to its size by

$$s_is_j' = s_is_j + \frac{s_is_j}{\sum s_is_j}\delta_i. \tag{7}$$

Symmetry is restored and then the process iterated in exactly the same way as before.

The smoothing given by equation (7) not only modifies each DEA according to its size, it also guarantees that no modified value is negative. After some simple algebra it can be shown that equation (7) simplifies to

$$s_is_j' = s_is_j\left(\frac{A_i}{\sum s_is_j}\right). \tag{8}$$

The results of using the improved smoothing algorithm on the test problem are shown in Table 2. The DEAs from the two end walls are now almost identical and the range of values for the DEAs from the other walls has been slightly reduced.

CONCLUSION

It is necessary to smooth approximate direct exchange areas to ensure that they are conservative. A modification to the smoothing algorithm of [2] has been presented. It has been shown that this modified algorithm produces better results than the original algorithm because it takes into account the differences in size of the approximate DEAs.

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Table 2. DEAs from the ends and the other walls to the pipe

	End 1	End 2	Wall 1	Wall 2	Wall 3	Wall 4
Original	0.0300	0.0306	0.0337	0.0355	0.0345	0.0360
Smoothed	0.0301	0.0302	0.0338	0.0356	0.0344	0.0358